Application of Sensitivity Coefficients to Parameter Estimation and Intrinsic Verification

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In parameter estimation, sensitivity coefficients play a key role. The sensitivity coefficient is $\partial\eta / \partial\beta_i$ where η is a dependent variable such as temperature and β_i is a parameter such as thermal conductivity. The scaled sensitivity coefficient, $\beta_i \partial\eta / \partial\beta_i$, is often more helpful than $\partial\eta / \partial\beta_i$. For example, it is desired that the scaled sensitivity coefficients in a given problem be both large and uncorrelated. Indeed an identifiability condition for estimation of *p* parameters is that the linear dependence condition of [1]

$$C_1\beta_1 \frac{\partial\eta}{\partial\beta_1} + C_2\beta_2 \frac{\partial\eta}{\partial\beta_2} + \cdots + C_p\beta_p \frac{\partial\eta}{\partial\beta_p} = 0 \qquad (1)$$

be <u>false</u>. Not all the C_i coefficients can be equal to zero. Equation (1) cannot be valid when all *p* of the parameters are to be estimated simultaneously.

If eq. (1) is equal to a non-zero term such as $-\eta$, then it might be possible to estimate simultaneously each of the *p* parameters. It is possible to find such a relation in many cases. Consider the transient heat conduction problem of two layers,

$$k_i \frac{\partial^2 T_i}{\partial x^2} = C_i \frac{\partial T_i}{\partial t},\ i = 1, 2;\ \ -k_1 \frac{\partial T_1}{\partial x}(0,t) = q_0 f(t),\ T_2(L_1 + L_2, t) = 0 \qquad (2)$$

with an initial temperature of zero and perfect contact between the layers; the heat flux $q_0 f(t)$ is known. The parameters are $k_1,\ k_2,\ C_1,\ C_2$. It can be shown that

$$k_1 \frac{\partial T_i}{\partial k_1} + k_2 \frac{\partial T_i}{\partial k_2} + C_1 \frac{\partial T_i}{\partial C_1} + C_2 \frac{\partial T_i}{\partial C_2} + T_i = 0 \qquad (3)$$

for $0 < x < L_1 + L_2,\ t > 0$. This summation identity has some similarities with eq. (1).

What can we learn from the identity given by eq. (3)? Several observations can be made including the following. First, with appropriate temperature measurements, it is possible to simultaneously to estimate all four parameters, $k_1, \cdots, C_2$, since the linear dependence condition given by eq. (1) is not satisfied. Second, the sum of the scaled sensitivity coefficients must be negative if the temperature *T* is positive; hence in this case the coefficients will tend to be negative. Third, the scaled sensitivities tend to be larger when *T* is larger; hence for an optimal experiment over a given temperature range the heat flux $q_0 f(t)$ is best chosen to be one that causes a rapid rise of the temperature a *x* = 0 to the maximum *T*. That would be much better than one which increases linear in time causing the maximum temperature to occur at the final time. Fourth, programming of any numerical or analytical solution of the problem given by eq. (2) must yield results which satisfy eq. (3); we call this an indication of intrinsic verification. Other points will be made in the talk.

Reference

J.V. Beck and K.J. Arnold, *Parameter Estimation in Engineering and Science,* John Wiley & Sons, New York, 1977, p. 22.